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LETTER TO THE EDTIOR

Universal local dynamics

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Abstract. We suggest that, near a critical point, the statistical behaviour of local variables averaged over times long on microscopic scales is universal, reflecting large scale spatial structure. The contention is supported with Monte Carlo studies of two-dimensional scalar models.

In the vicinity of a continuous phase transition the long wavelength fluctuations in the ordering field exhibit behaviour which, modulo trivial scale factors, is insensitive to local details. This is the well known phenomenon of universality. To expose and study the universal physics it is customary to implement some form of spatial coarse graining of the ordering field and to study the flow of the coupling constants of the field theory (Wilson and Kogut 1974), or the flow of the configurations themselves, under the action of this coarse graining (Bruce 1981, Binder 1981). In this letter we show that the universal physics may alternatively be exposed by a temporal coarse graining of the field: the statistical behaviour of local variables averaged over times long enough on 'microscopic' scales is itself uniquely determined by, and representative of, a universality class. The underlying idea is straightforward: the time-averaging procedure discriminates against fast fluctuations so that the behaviour of the time coarse grained variable reflects the slowest modes which are the long wavelength modes. The idea is not new: it has been discussed qualitatively in the context of electron paramagnetic resonance (EPR) line shapes near phase transitions (Bruce *et al* 1979). Here the idea is tested in a quantitative way. The results extend the fractal configurational picture of equilibrium behaviour (Bruce and Wallace 1983) to embrace time-dependent phenomena and confirm that suitable (time coarse graining) local probes may indeed access universal configurational structure.

Consider a system exhibiting a continuous phase transition. In what follows we suppose that the system has a scalar ordering field, defined by a set of local coordinates $\{\phi\}$ associated with the sites, i, of a d-dimensional lattice. We shall also restrict attention to the situation precisely at the critical point. For reference, we first review briefly the now well documented claims regarding spatial coarse graining, summarised in the equations

$$M_L = L^{-d} \sum_i \phi_i \tag{1a}$$

$$P(M_L) = L^{\lambda_L} P_L^*(aL^{\lambda_L} M_L) \tag{1b}$$

$$\lambda_L = (d-2+\eta)/2. \tag{1c}$$

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Equation (1*a*) defines the spatially coarse grained field ('magnetisation') M_L as the average of the local variable over the L^d sites i within a block of side L. Equation (1*b*) asserts that, for large enough coarse graining lengths, L (and modulo a non-universal scale factor a) the spatially coarse grained field is characterised by a universal distribution, P_L^*. Equation (1*c*) indicates that the statistical self-similarity of the coarse grained field, viewed with different coarse graining lengths, is secured by a field scaling whose L dependence is defined by the 'anomalous dimension', η, of the ordering field. The form of the distribution (1*b*) has been studied explicitly in a number of cases (Binder 1981, Bruce 1981, Brown 1986). In particular, for members of the $d=2$ Ising universality class, the distribution has a strongly double-peaked structure characterised by a fixed point value of the cumulant ratio $G_L=(3-\langle M_L^4\rangle/\langle M_L^2\rangle^2)/2$ of $G_L^*\simeq 0.84$.

Now consider the time coarse graining procedure. The analogues of equations (1) are

$$M_\tau=\tau^{-1}\int_0^\tau \phi(t)\,\mathrm{d}t \tag{2a}$$

$$P(M_\tau)=\tau^{\lambda_\tau}P_\tau^*(b\tau^{\lambda_\tau}M_\tau) \tag{2b}$$

$$\lambda_\tau=\lambda_L/z. \tag{2c}$$

Equation (2*a*) defines the time coarse grained field M_τ through the time integral of the local field ϕ (the site index i has been suppressed) over a coarse graining time τ. Equation (2*b*) asserts the universality (modulo a scale factor b) of the distribution of the variables, for coarse graining times long enough on microscopic scales. Equation (2*c*) indicates that the self-similarity of the time coarse grained configurations emerges through a field scaling whose τ dependence reflects the dynamic critical index z. These predictions follow from dynamic scaling arguments, at the core of which is the recognition that, for large enough τ, the moments $\langle M_\tau^n\rangle$ are dominated by the fluctuations of a small wavevector.

To test the validity of these claims and to probe the form of the fixed point distribution (2*b*) we have made Monte Carlo studies of the spin-$\frac{1}{2}$ and spin-1 $d=2$ Ising models (with non-conserved order parameter) which are believed to belong to the same universality class. The MC procedure employed the standard Metropolis algorithm, implemented within a parallel framework on the Edinburgh Distributed Array Processors. The claim that this pseudodynamics does indeed fall into the intended universality class has been investigated elsewhere (Williams 1985). Our studies have been conducted at the respective bulk critical temperatures of the models; they are thus susceptible to finite-size effects. We have studied various sytem sizes, L_0; here we present data accumulated on systems of side $L_0=128$.

Figure 1 shows the behaviour of the cumulant ratio $G_\tau=(3-\langle M_\tau^4\rangle/\langle M_\tau^2\rangle^2)/2$ for spin-1 and spin-$\frac{1}{2}$ models. For small coarse graining intervals the ratios reflect system-specific local details and are quite different (the $\tau\to 0$ limit of G_τ is $G_\tau=1$ for spin $\frac{1}{2}$ and $G_\tau=0.905$ for spin 1 in accordance with the equilibrium distributions of the local variables in the two cases). For large τ, however, the two cumulants do indeed approach one another. In this regime the associated G_τ value is not quite constant, implying that the configurations are not quite timescale invariant. This is a manifestation of the finite system size, whose effects, even at a spatially local level, are inevitably amplified into signficance by the coarse graining procedure. To accommodate these effects we note that, according to finite-size scaling, one must expect that in the limit of large τ the cumulant ratio will become a universal scaling function $\tilde{G}(\tau/\tau_0)$, where τ_0 defines

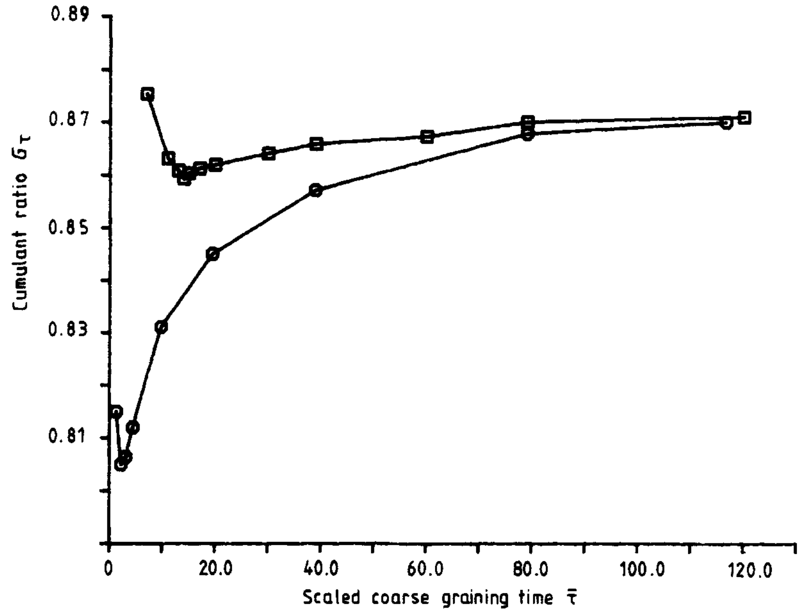

Figure 1. The cumulant ratio G_τ, defined in the text, for spin-$\frac{1}{2}$ (□) and spin-1 (○) systems of side $L = 128$, at their respective bulk critical points, and plotted against the scaled coarse graining time $\bar{\tau}$. The statistical uncertainties do not exceed the symbol sizes. The lines are merely to guide the eye.

the 'outer' timescale set by the finite relaxation time of the macroscopic magnetisation ($\tau_0 = cL_0^z$, with c a non-universal constant). With this in mind, we have chosen in figure 1 to plot the cumulant against the scaled time $\bar{\tau}$ defined by $\bar{\tau} = \tau$ (in MC steps per site) for spin $\frac{1}{2}$ and $\bar{\tau} = \tau(\tau_0^{(1/2)}/\tau_0^{(1)})$ for spin 1. The relaxation time ratio $\tau_0^{(1/2)}/\tau_0^{(1)} \simeq 0.313$ was determined by independent studies of the autocorrelation function for the two models. The large scaled-time data collapse shown in figure 1 bears out these expectations. This data collapse is expressed in a more evocative fashion in figure 2, which shows the distributions of the variables M_τ for the two models, evaluated for the same scaled time $\bar{\tau} = 79$. The distributions have been normalised to the same variance to absorb the difference in the non-universal scale factors, b (equation (2b)). The accord is striking, as is the similarity of the distribution to that of the space coarse grained variables in the critical, scale-invariant, limit (Binder 1981). Indeed a finite-size scaling analysis indicates for the bulk fixed point $G_\tau^* = \tilde{G}_\tau(\bar{\tau} \simeq 0)$ a value $G_\tau^* = 0.84 \pm 0.01$, remarkably close to G_L^*.

Although the analytic scaling theory we have developed in conjunction with this work offers no reason to believe that this correspondence is exact, the result strongly substantiates our basic contention, that the time coarse grained variables reflect the same universal large length scale structure as that exposed by spatial coarse graining. In particular, in the two-dimensional case studied here, it is natural to regard the form of the distribution (figure 2) and the fractal time profile of the time coarse grained variable, implied by the scaling form (2b), as a manifestation of the nested droplets, known to account well for the equilibrium properties (Bruce and Wallace 1983). At a more practical level our results suggest that local probes such as EPR, which through motional narrowing average out fast fluctuations, can expose universal large length scale structure. Indeed the EPR lineshape in the slow motion regime can be directly

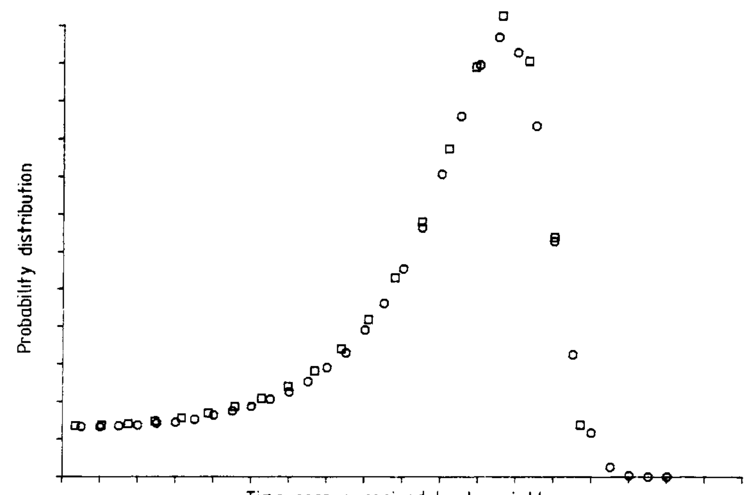

Figure 2. The probability distributions of the time coarse grained variables M_τ for spin-$\frac{1}{2}$ (□) and spin-1 (○) models at criticality, evaluated for scaled coarse graining time $\bar{\tau} = 79$. In each case the variable has been scaled to have unit variance. The distributions are symmetric about $M_\tau = 0$.

related to the fixed point distribution P_τ^*. Our results also suggest the viability of a new computational scheme for MC studies of critical dynamics, complementary to those (utilising spatial coarse graining) currently available (Tobochnik *et al* 1981), based on the time scaling properties of the distribution (2b) and its moments, and analogous to that developed by Binder (1981) for equilibrium phenomena. These points will be developed in subsequent work.

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